Theory of breakup and fusion of weakly bound nuclei

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Abstract

We assess the validity of the semiclassical approximation of Alder and Winther in the study of breakup and fusion reactions induced by weakly bound projectiles. For this purpose, we compare semiclassical results with results of full quantum mechanics calculations. We show that the semiclassical method leads to accurate results for the breakup cross section. We then adopt a semiclassical approximation for the $l$-dependent fusion probabilities and evaluate the cross section in a schematic two-channel problem. In this case, the semiclassical results are accurate above the Coulomb barrier but cannot reproduce the enhancement of the fusion cross section at sub-barrier energies. We show that this shortcoming can be eliminated through an analytical continuation of the time variable.

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I. INTRODUCTION

The study of nuclear reactions in collisions of weakly bound nuclei has attracted considerable interest in the last decades [1]. In particular, several measurements of fusion and breakup cross sections in reactions induced by stable [2] and radioactive weakly bound nuclei have recently been made [2]. These new data call for adequate theoretical tools for their interpretations.

The first estimates of the complete fusion cross section for weakly bound projectiles lead to conflicting results. While some calculations predicted a suppression of this cross section [3], others predicted its enhancement [4]. In both cases, however, the calculations were quite schematic in their inclusion of the breakup channel. It was only recently that more realistic coupled-channels calculations became possible [5, 6]. These calculations employed the Continuum Discretized Coupled-Channel (CDCC) method, which, although being the proper way to describe coupled-channels problems involving the continuum, makes the calculations more complicated. Here we review our recent work based on the semiclassical theory of Alder and Winther (AW) [7], as follows. In the next section, we describe the semiclassical method and derive expression for the cross sections. In section III, we show how the method can be extended to coupled channel problems involving the continuum and review its application to the study of the breakup of weakly bound projectiles. In section 4, we show, through schematic examples, how it can be employed to estimate the fusion cross section in collisions of weakly bound nuclei. In section 5 we present the conclusions of the present work.

II. THE SEMICLASSICAL COUPLED-CHANNELS EQUATIONS

In a coupled channel problem the system Hamiltonian can be written as

\[ H = H_0(r) + h(\xi) + V(\xi, r). \]  

Above, \( r \) is the vector joining the centers of the target and the projectile nuclei and \( \xi \) stands for all relevant intrinsic coordinates. The Hamiltonian contains three terms. The first one, \( H_0 \), is the sum of the relative kinetic energy operator with the optical potential. The second term is the intrinsic hamiltonian, \( h \), which has eigen-vectors \( \varphi_\alpha \) and eigenvalues \( \varepsilon_\alpha \). The third one, \( V(\xi, r) \), couples the relative and intrinsic degrees of freedom.
The method developed by Alder and Winther (AW) for the study of Coulomb excitation consists of treating the relative motion by classical mechanics whereas the intrinsic motion is handled as a time-dependent quantum perturbation problem. Solving the classical equations of motion with the Hamiltonian $H_0$ for a given impact parameter $b$ and incident CM energy $E$, the classical trajectory $r_b(t)$ is determined. The internal wave function for the excitable nucleus is then found by solving the Schrödinger equation for the time dependent Hamiltonian

$$H(\xi, t) = h(\xi) + V(\xi, r_b(t)) \equiv h(\xi) + V(\xi, t).$$

i.e.,

$$H(\xi, t) \psi(\xi, t) = i\hbar \frac{\partial \psi(\xi, t)}{\partial t}.$$  

Expanding $\psi(\xi, t)$ in terms of a properly truncated set of eigenfunctions of $h$, eq.(3) leads to the set of coupled differential equations

$$i\hbar \dot{a}_\alpha(t) = \sum_\beta V_{\alpha,\beta}(t) e^{i(\varepsilon_\alpha - \varepsilon_\beta)t/\hbar} a_\beta(t); \quad \alpha, \beta = 0, 1, ..., N.$$ 

In order to study the excitation process one should first determine the matrix elements $V_{\alpha,\beta}(t) = \int d\xi \varphi^*_\alpha(\xi) V(\xi, t) \varphi_\beta(\xi)$, where $\varphi^*_\alpha (\varphi_\beta)$ is the eigenstate of $h$ with eigenvalue $\varepsilon_\alpha (\varepsilon_\beta)$, and then solve the above equations with initial conditions corresponding to the ground state of the internal system,

$$a_\alpha(t \to -\infty) = \delta(\alpha, 1).$$

From the final values of these amplitudes one may determine, for the impact parameter considered, the probability of leaving the system in each of the excited states included in the expansion and consequently the corresponding cross sections. The integrated cross section for one of these channels, $\alpha$, is given by the integral over impact parameter

$$\sigma_\alpha = 2\pi \int P_\alpha(b) \ b \ db; \quad \text{with} \quad P_\alpha(b) = |a_\alpha(t \to \infty)|^2.$$ 

In a similar way, the angular distributions in channel $\alpha$ can be related to the final population of this channel through

$$\frac{d\sigma_\alpha(\theta)}{d\Omega} = \frac{d\sigma_{el}(\theta)}{d\Omega} P_\alpha(b_\theta),$$ 

Where $d\sigma_{el}(\theta)/d\Omega$ is the elastic cross section (which may be determined through, e.g. an optical model calculation) and $b_\theta$ is the impact parameter associated with the scattering
For each $l, j, m$, discretize the continuum in bins.

angle $\theta$ through the classical trajectory. In the case where the elastic cross section is approximated by its classical value, the above expression should be multiplied by a factor $A_{abs}(b_\theta)$, to account for the absorption along the classical trajectory.

III. APPLICATION OF THE AW METHOD TO BREAKUP REACTIONS

In this section we give a detailed description of the use of the AW method to calculate the cross section for breakup of a weakly bound projectile. This example is very important for our purposes, since the coupled equations appearing in this problem are the same as those for the fusion of weakly bound projectiles.

The AW method is particularly useful in problems involving a large number of coupled channels, where full quantum mechanical calculations become very complicated. This is the situation where the elastic channel is strongly coupled to breakup states. In this case, one of the collision partners, usually the projectile, breaks into two or more fragments, $F_1, F_2, \ldots$ moving in the continuum. The channel label is then continuous and the problem becomes very complicated. This difficulty is usually handled by the CDCC method [8], where the continuum is discretized in a set of bins of variable size. The CDCC calculations describing the breakup of a projectile $P$ are performed replacing the continuum by a finite number of configurations of the $P = F_1 + F_2 + \ldots$ system [9]. The energies of these internal excitations are assumed to extend to some maximum energy, characterized by the relative wave number $q_{\text{max}}$. The energy range is then divided in intervals $\Delta q_i \equiv [q_{i-1}, q_i], i = 1, \ldots, N, q_0 = 0,$
\[ q_N = q_{\text{max}}. \] Each of these bins is labeled by \( \alpha \equiv (i, J_p, J_{F_1}, J_{F_2}, l). \) From the radial wave functions \( \phi_\alpha(q, x) \), representing the scattering of the fragments, we construct a set of superposition radial wave functions associated to each bin,

\[
u_\alpha(x) = \int_{q_{i-1}}^{q_i} g_\alpha(q) \phi_\alpha(q, x) dq , \tag{8}
\]

where the \( g_\alpha \) are appropriately selected weight functions for the construction of each bin. It has been verified that results of the calculations are not much dependent on this choice. This procedure is schematically represented in figure 1. In the case of bound states, \( u_\alpha(x) \) coincides with the radial eigenfunction of \( h \).

Recently this discretization procedure has been used in the realistic calculations of Nunes and Thompson [5] to study \(^8\text{B} \) breakup in the \(^8\text{B} + ^{58}\text{Ni} \) collision, at \( E_{\text{lab}} = 26 \text{ MeV} \). The resulting angular distribution is represented by solid circles in figure 2. The authors have pointed out that continuum-continuum couplings play a very important role in the calculations.

Marta et al. [10] studied the same problem with the Alder-Winther method. They discretized the continuum using the same bins and angular momentum states as in ref.
 Their results correspond to the solid line in figure 2. The agreement with the CDCC calculations is very good. This suggests that the semiclassical method may be an important tool to study nuclear reactions induced by weakly bound projectiles.

IV. EXTENSION TO FUSION REACTIONS

In a coupled channel calculation, the equations are usually expanded in partial-waves and the fusion cross section is expressed in terms of the quantities

\[ P_l^F(\alpha) = \frac{4k}{E} \int dr \ |u_{al}(k_\alpha, r)|^2 W_{\alpha}^F(r), \tag{9} \]

which represents the fusion probability in channel \( \alpha \) at the \( l^{th} \) partial wave. In eq.\( (9) \), \( u_{al}(k_\alpha, r) \) is the radial wave function and \( W_{\alpha}^F(r) \) is the imaginary potential accounting for fusion absorption in channel \( \alpha \).

An extension of the AW theory to the case of fusion reactions has been recently proposed in ref. [11]. The basic idea is to approximate the fusion probabilities \( P_l^F \) by

\[ P_l^F(\alpha) \simeq \bar{P}_l^{(\alpha)} T_l^{(\alpha)}(E_\alpha). \tag{10} \]

In the above expression \( T_l^{(\alpha)}(E_\alpha) \) is the probability that a particle with energy \( E_\alpha = E - \varepsilon_\alpha \) and reduced mass \( \mu_\alpha = m_0 A_P A_T / (A_P + A_T) \) tunnels through the potential barrier in channel \( \alpha \), and \( \bar{P}_l^{(\alpha)} \) represents the probability that the system is found in that channel at the point of closest approach on the classical trajectory. In ref. [11] this method, under simplifying assumptions, was used to evaluate the complete (CF) and the incomplete fusion (ICF) cross sections in reactions induced by weakly bound projectiles. In those calculations the ground state was assumed to be the only bound state of the projectile, with breakup processes producing just two fragments, \( F_1 \) and \( F_2 \). In this way, the labels \( \alpha = 0 \) and \( \alpha \neq 0 \) correspond to the GS and the breakup states, respectively, these last represented by two unbound fragments. If one neglects sequential fusion contributions to the CF process (an assumption that recent classical calculations by Hagino and collaborators do not completely support [13], complete fusion could only arise from the elastic channel. Thus the sum of channel contributions is reduced to a single term with

\[ \bar{P}_l^{(0)} \equiv P_l^{\text{surv}} = |a_0(t_{c.a.})|^2, \tag{11} \]
where the amplitude $a_0$ is evaluated along a trajectory with impact parameter $b = l/k$. The factor $P_l^{\text{surv}}$ is usually called survival (to breakup) probability. Therefore

$$
\sigma_{\text{CF}} = \frac{\pi}{k^2} \sum_l (2l + 1) P_l^{\text{surv}} T_l^{(0)}(E).$$

(12)

The accuracy of this procedure was checked in a preliminary two-channel calculation for the scattering of $^6$He projectiles on a $^{238}$U target at near barrier energies [11]. The weakly bound $^6$He nucleus dissociates into $^4$He and two neutrons, with threshold energy $B = 0.975$ MeV. The elastic channel is strongly coupled to the breakup channel and the influence of this coupling on the fusion cross section is quite important. In this case the breakup channel is represented by a single effective state [12]. For simplicity, the effective channel was treated as a bound state but it was assumed to contribute only to incomplete fusion. In this way, the CF cross section was given by eq.(12). The threshold energy was neglected and the same potential barrier was used for both channels. The optical potential was given by Woods-Saxon parametrizations with $V_0 = -60$ MeV, $r_{0r} = 1.25$ F, $a_r = 0.65$ F, $W_0 = -50$ MeV, $r_{0i} = 1.0$ F and $a_i = 0.1$ F. The form factor had the radial dependence of the electric dipole coupling with an arbitrary strength chosen in such a way that the coupling modifies the cross section of the one dimension penetration barrier appreciably. The CF cross section was shown to be in very good agreement with the results of a full coupled-channels calculation at above barrier energies. However, in that work the agreement obtained at sub-barrier energies was very poor. As illustrated in fig. 3, the semiclassical calculation drastically underestimates the CF cross section in that energy region. Figure 3 also indicates that near the Coulomb barrier the semiclassical cross section fluctuates with the collision energy. This is a consequence of orbiting. When the energy approaches the barrier height for one partial-wave, the collision time becomes very large and changes rapidly with the collision energy. Since the sistem remains a very long time in the vicinity of the barrier radius, where channel coupling is very strong, the populations of the two channels oscillate several times. The final population in the elastic channel, which determines the CF cross section, assumes an essentially random value between zero and one. This shortcoming tends to disappear as the elastic channel couples to a large number of continuum states, also coupled among each other. In this way, the elastic channel loses amplitude in an irreversible way. This situation is simulated below, by a two-channel model with complex excitation energy.

In order to improve the semiclassical model at sub-barrier energies, one can resort to
FIG. 3: Comparison between the Coupled Channels (full line) and the semiclassical (dots) calculations of the fusion cross section of ref. [11].

the analytical continuation method, which consists of introducing the imaginary part of the time variable to obtain a classical trajectory in the sub-barrier region [15]. This procedure is illustrated in figure 4, where the time scale is chosen such that $t = 0$ at the external turning point, $r_e$. Along the incident branch of the trajectory (A), the time develops on the real axis as the system approaches the barrier. At $r = r_e$, the trajectory splits into two parts: the reflected branch (B), and the classically forbidden transmission branch, (C). On the former, which is not relevant to the fusion process, the time remains real. Along branch (C) the real part of the time remains equal to zero while its imaginary part becomes increasingly negative, until the trajectory reaches its exit point $r_i$, at $t = -i\Delta$. This trajectory is then continued into the internal classically allowed region (D), and into the strong absorption radius, $R_F$, where fusion takes place. Over this branch, the real part of $t$ grows whereas its imaginary part remains constant, $t_I = -\Delta$. The fusion probability is then evaluated in terms of the elastic Alder-Winther amplitude calculated along the trajectory $A - C - D$. The survival probabilities become

$$P_I^{\text{surv}} = |a_0(t_F)|^2,$$  \hspace{1cm} (13)

where $t_F$ is the complex value of the time variable at which the system reaches the strong absorption radius.
FIG. 4: Analytical continuation of the time variable. The upper panel shows the branches of the classical trajectory and the lower panel the evolution on the complex time plane.

In order to account for the excitation energy in the breakup channel and simulate the irreversible nature of the breakup process [16], a complex value $E = Q - i\Gamma/2$ was assigned to the energy of the effective channel. This assumption is based on the fact that an exponentially decaying state with mean life $\tau = \hbar/\Gamma$, may be obtained through the inclusion of a constant imaginary potential equal to $-i\Gamma/2$ in the system Hamiltonian. However, this procedure requires some care. Solving the AW equations does not present difficulties since the population of the resonant state is vanishingly small as $t \to -\infty$, but, on the other hand, the numerical solution of the coupled-channel equations, requires additional attention. In order to handle this problem one must switch-off the $-i\Gamma/2$ contribution to the imaginary potential at some distance much larger than the range of the potentials, and then match the radial wave functions with their asymptotic forms [14], checking for the constancy of the results with the variation of the switch-off distance.
FIG. 5: Quantum mechanical (full line) and semiclassical (full circles) CF cross sections for $B = 0.975$ MeV and $\Gamma = 2$ MeV. At sub-barrier energies, the contribution from the elastic channel alone is already as large as the cross section in the no-coupling limit (dashed line). This is a consequence of channel coupling.

The CF cross sections obtained with the above discussed procedures are shown as solid circles in figure 5, where they are compared with results of the CC method (solid line) and also with a calculation performed in the no-coupling limit. Through this comparison we conclude that with these improvements the semiclassical model is able to provide accurate results both above and below the Coulomb barrier.

V. CONCLUSIONS

We have considered the application of the semiclassical method of Alder and Winther to breakup and fusion processes induced by weakly bound nuclear beams. In the case of breakup, we calculated the $^8$B breakup cross section in collisions with a $^{58}$Ni target using a continuum discretized basis for the breakup channel. The results are very close to those of a fully quantum mechanical calculation that considers the same set of continuum states.

In the case of fusion, CF cross sections were evaluated in a schematic two-channel model,
in which the breakup states were pooled into a single effective channel. Previous calculations [11] have shown the need to improve the semiclassical description of that process at under-barrier energies. In order to describe the fusion cross section at sub-barrier energies we found necessary to perform an analytical continuation of the time to complex values. Through this procedure, the comparison of the semiclassical calculations with results of a fully quantum mechanical calculation indicate that the semiclassical method is able to provide accurate results, both above and below the Coulomb barrier.

One should remark that the semiclassical method discussed in this paper has the advantages of employing the correct barriers for incomplete fusion and to open for the possibility of including the sequential fusion mechanism into the calculation of the CF cross section. More quantitative calculations employing a less schematic discretization of the continuum are presently under development.

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